List of talks and abstracts

OAK in Metz, July 8-10, 2025

June 18, 2025

1 Plenary talks

- Ilka Agricola (Marburg, Germany) : *On the (non-)existence of complex structures on S^6 - the Hopf problem*
- Alessandra Frabetti (Lyon, France) : *Poisson bundles over unordered configurations*
- Fréderic Hélein (Paris, France) : *Kaluza-Klein theories without a priori fibration hypotheses*
- Taro Kimura (Dijon, France) : *Wall-crossing in geometric representation theory*
- Manuel de León (Madrid, Spain) : *Multicontact geometry and action-dependent Field Theories*
- Camille Laurent-Gengoux (Metz, France) : *Making singular foliations look better*
- Robert Yuncken (Metz, France) : *Smoothness of solutions of PDEs and tangent groupoids*

2 PhD students' talks

- Hai Chau Nguyen (Lyon, France) : *Monoidal Structures of Cone Bundles over Multiconfiguration Orbifold Spaces*
- Lorenzo Scaglione (Lyon, France) : *Bulk edge correspondence for aperiodic tight binding operators : relating the integrated density of states to forces*

- Ouneïs Gloton (Dijon, France) : *Instantons in supergroup gauge theory*
- Dimitrios Makris (Dijon, France) : *Bihamiltonian structures of the genus-zero Whitham hierarchy*
- Maxime Wagner (Metz, France) : *Dynamique Hamiltonienne sur des variétés multisymplectiques*

3 Abstracts of the plenary talks

- Ilka Agricola (Marburg, Germany) : *On the (non-)existence of complex structures on S^6 - the Hopf problem*
  In this review talk, I will outline the history of the problem of proving or disproving the existence of complex structures on S^6 and explain why this question is interesting. It goes back to a 1947 paper by Heinz Hopf. Moreover, I will explain the rich structure of S^6 – as an almost complex manifold, and how it relates to the exceptional Lie group G_2, octonions, and spin geometry.
- Alessandra Frabetti (Lyon, France) : *Poisson bundles over unordered configurations*
  In this talk we explain how to construct a Poisson algebra bundle whose distributional sections are suitable to represent multilocal observables in classical field theory. To do this, we work with vector bundles over the unordered configuration space of a manifold M and consider the structure of a 2-monoidal category given by the usual (Hadamard) tensor product of bundles and a new (Cauchy) tensor product which provides a symmetrized version of the usual external tensor product of vector bundles on M. We use the symmetric algebras with respect to both products to obtain a Poisson 2-algebra bundle mimicking the construction of Peierls bracket from the causal propagator in field theory.
- Fréderic Hélein (Paris, France) : *Kaluza-Klein theories without a priori fibration hypotheses*
  I will present a Lagrangian action on fields, the critical points of which lead to solutions of the Einstein-Yang-Mills equations, in the spirit of Kaluza-Klein

theories. The novelty is that the a priori fiber bundle structure hypothesis is not required: fields are defined on a "space-time" Y of dimension $4+r$ without any a priori principal bundle structure, where r is the dimension of the structure group. If the latter group is compact and simply connected, to each solution of the Euler-Lagrange equations it corresponds a 4-dimensional pseudo-Riemannian manifold X (which can be interpreted as our usual space-time) in such a way that Y acquires a principal bundle structure over X equipped with a connection. Moreover the metric on X and the connection on Y are solutions of the Einstein-Yang-Mills system. If the structure group is $U(1)$ (the case which corresponds to the Einstein-Maxwell system) the situation is slightly degenerated and supplementary hypotheses are necessary.

- Taro Kimura (Dijon, France) : *Wall-crossing in geometric representation theory*
  The wall crossing phenomena refers to discontinuous changes of certain quantities as one goes across codimension-one walls in a parameter space, which is typically identified as a moduli space. In this talk, I'll start with elementary examples of the wall crossing formula and then discuss its interpretation in the context of geometric representation theory together with its application to (quantum) KZ equations and integrable systems.

- Manuel de León (Madrid, Spain) : *Multicontact geometry and action-dependent Field Theories*
  Action-dependent field theories are systems where the Lagrangian or Hamiltonian depends on new variables that encode the action. They model a larger class of field theories, including non-conservative behavior, while maintaining a well-defined notion of symmetries and a Noether theorem. This makes them especially suited for open systems. After a conceptual introduction, a quick presentation of a new mathematical framework is made for action-dependent field theory: multicontact geometry. We also introduce a graded bracket of forms on multicontact manifolds. This bracket satisfies a graded Jacobi identity as well as two different versions of the Leibniz rule, one of them being a weak Leibniz rule, extending the well-known notions in contact geometry. The Jacobi bracket also permits to study the evolution of observables and the dissipation phenomena, which we also address. Finally, we apply the results to classical dissipative field theories.

- Camille Laurent-Gengoux (Metz, France) : *Making singular foliations look better*

In this lecture I will explain a description given by my former Ph-student Ruben Louis in his PHD of a method of resolution of singularities due to Omar Mohsen. More precisely, I will explain why it matches with a (forgotten) notion of resolution of singularities of coherent sheaves, and why it matters. I will then explain the outline of a construction of an infinity-groupoid that extends Androulidakis-Skandalis holonomy groupoid, also in collaboration with Ruben Louis.

- Robert Yuncken (Metz, France) : *Smoothness of solutions of PDEs and tangent groupoids*
  Alain Connes realized that many features of the pseudodifferential calculus for elliptic differential operators can be understood via the notion of a tangent groupoid. Debord and Skandalis showed that the tangent groupoid can be used to define the pseudodifferential calculus. There have followed a number of results using more elaborate analogues of Connes' tangent groupoid to prove regularity properties of subelliptic PDEs, including a proof of the Helffer-Nourrigat conjecture concerning operators built from bracket-generating vector fields. We will describe these developments, beginning with a description of the problems and the construction of the groupoids.

4 Abstracts of the PhD students' talks talks

- Hai Chau Nguyen (Lyon, France) : *Monoidal Structures of Cone Bundles over Multiconfiguration Orbifold Spaces*
  In this talk, we present a categorical framework for describing the algebra of (off-shell) observables in covariant (non-scalar) field theories. Specifically, we model these observables as kernels represented by sections of vector bundles–more precisely, cone bundles–over the multiconfiguration space of a spacetime manifold.

  We begin by characterizing the multiconfiguration space of a manifold as an orbifold, a type of singular space that naturally arises in this context. We then introduce the notion of bundles over such orbifolds, highlighting two key

monoidal structures: the Hadamard tensor product, which encodes local polynomial observables, and the Cauchy tensor product, corresponding to multilocal polynomial observables.

We will share both established results and ongoing developments within this framework. This work is part of a PhD project under the supervision of Alessandra Frabetti and Leonid Ryvkin.

- Lorenzo Scaglione (Lyon, France) : *Bulk edge correspondence for aperiodic tight binding operators : relating the integrated density of states to forces*
  We consider a particular class of 1D aperiodic models with the aim to understand how their internal degrees of freedom contribute to their topological invariants and the possible relations (correspondences) among them. Our approach is based on the well-established C*-algebraic approach to solid state physics and the description of topological invariants by K-theory. In this talk, we will establish a bulk-edge correspondence relating bulk invariants to edge ones, giving them a physical interpretation. In our case, the invariants correspond to the Integrated Density of States for the bulk system, while the invariants of the edge system correspond to some spectral flows traversing the spectral gaps of the Hamiltonian. We apply then our abstract results to a simple model, the Kohomoto model. Numerical simulations allow us to give a concrete illustration of the theorems.

- Ouneïs Gloton (Dijon, France) : *Instantons in supergroup gauge theory*
  The goal of this talk is to give a framework for defining and studying gauge theories for a structure Lie supergroup G, in the setting of supermanifold geometry. A model of interest is Yang-Mills theory over an oriented closed Riemannian 4-manifold, with $G = U(n_0|n_1)$ the super unitary group. We then define instantons in this setting and their moduli problem, and give the state of our research on the case $M = S^4$ and $G = U(2|2)$, as well as leads for representability of the moduli functor.

- Dimitrios Makris (Dijon, France) : *Bihamiltonian structures of the genus-zero Whitham hierarchy*
  The genus-zero Whitham hierarchy, originally introduced by Krichever in 1994, is a family of dispersionless evolutionary PDEs for an infinite number of dependent variables, which arises from the Whitham averaging method. It is an extension of the dispersionless KP hierarchy and is related, by reduction, to the dispersionless limit of several important integrable systems, such as the KdV, Gelfand-Dickey and constrained KP hierarchies. Using the R-matrix

techniques originally developed by Semenov-Tian-Shansky et al., we construct a family of bihamiltonian structures on the algebra of (m+1)-tuples of formal Laurent series, with an R-matrix associated with a splitting of this algebra in two subalgebras. By Dirac reduction, we obtain an infinite family, indexed by m+1 positive integers, of bihamiltonian structures for the Whitham hierarchy. Finally, we show how these bihamiltonian structures correspond to the flat metric and intersection form of a recently introduced family of infinite-dimensional Frobenius manifolds, thereby providing an explicit bihamiltonian formulation of their principal hierarchies.

- Maxime Wagner (Metz, France) : *Dynamique Hamiltonienne sur des variétés multisymplectiques*
  L'étude de la dynamique hamiltonienne en gómétrie multisymplectique est vaste. Cet exposé aura pour but d'étudier différents cas rencontrés majoritairement en géométrie 2-plectique, avec ou sans coordonnées de Darboux. Des équations aux dérivées partielles classiques seront retrouvées ; nous verrons l'existence de solutions à des classes d'EDP totalement non-linéaires et certaines solutions seront exhibées ; enfin, l'étude des solutions de dimension une et deux sur la sphère hexadimensionnelle viendra clore cette présentation.