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OAK2 in Metz, June 22-25, 2026

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## 3 Abstracts of the plenary talks

- Mélanie Bertelson (Bruxelles, Belgique)  
*Introduction to locally conformal symplectic manifolds and a corresponding "h-principle"*  
The purpose of this talk is to introduce locally conformal symplectic manifolds and explain the ideas and main ingredients of the proof of a h-principle for those structures obtained in a joint work with Gaël Meigniez.

- Christian Blohmann (Bonn, Germany)

*Cartan calculus and multisymplectic geometry on field spaces*

I will show how to construct a Cartan calculus abstractly on an object in a tangent category. The guiding example is a field space, that is, the set of sections of a smooth fiber bundle with its functional diffeology. By construction, its Cartan calculus preserves smooth homotopies but sidesteps all functional analytic issues. I will explain how easily multisymplectic geometry and homotopy momentum maps carry over to this framework.

- Véronique Chloup and Angela Gammella-Mathieu (Metz, France)

*Actions of Lie 2-algebras and comomentum maps for 2-plectic manifolds*

A way to encode Hamiltonian mechanics of fields theories is to replace symplectic manifolds by multisymplectic manifolds (that is  $n$ -plectic manifolds with  $n > 1$ ). Traditionally, for a  $n$ -plectic manifold, the higher analog of the Poisson algebra of observables associated to a symplectic manifold is the Lie  $n$ -algebra of Carl Rogers.

Here, following ideas of Nestor Leon Delgado, we introduce the notion of an action (a 2-action) of a Lie 2-algebra  $\mathfrak{g}$  on a manifold  $M$  generalising the infinitesimal action of a Lie algebra. Then, considering a 2-plectic structure on  $M$  (which is the simplest case of multisymplectic structure), we define a 2-comomentum map on  $M$  as a lift of the 2-action (from  $\mathfrak{g}$ ). We explain how we are led to enlarge the usual Lie 2-algebra of observables of Carl Rogers, in order to make the 2-action effective. We then give a cohomological characterisation of our objects via a generalised double complex, which extends the related results of symplectic and 2-plectic geometry.

Our talk is based on joint work with Philippe Bonneau and Tilmann Wurzbacher.

- Jordi Gaset (Madrid, Spain)

*Towards a classification of field theories: a multisymplectic Arnold-Liouville theorem*

I will present a program for the classification of field theories based on factorization. The proposal builds upon recent results comparing different geometric structures. The factorization scheme I will put forward extends classical and recent findings in symmetry reduction. Moreover, it formalizes in a geometric language new ideas from theoretical physics. If time permits, I will show how the program can contribute to addressing open problems in theoretical physics. Against this background, I will present a consequential result: a covariant Arnold-Liouville theorem for multisymplectic systems. This theorem provides a powerful semi-local characterization of integrable field theories. Moreover, it

sheds light on the basic blocks of the said classification program.

- François Gières (Lyon, France)

*On the inverse problem of variational calculus*

We review different aspects of (and approaches to) the inverse problem of variational calculus, i.e. the questions of existence, uniqueness, construction, ... of a Lagrangian (action integral) for a given system of ordinary or partial differential equations.

- Simone Gutt (Bruxelles, Belgique)

*Symplectic connections*

A symplectic connection on a symplectic manifold is a linear torsion free connection for which the symplectic 2-form is parallel. Symplectic connections are intimately linked to deformation quantization: any natural star product defines a unique symplectic connection and any symplectic connection allows the Fedosov-type-construction of natural star products characterized by some extra data. When there is a group acting on the manifold, preservation of a given natural star product implies that the group acts by symplectomorphisms preserving the associated connection and the extra data and vice versa. Symplectic connections always exist, but of course invariant symplectic connections do not.

On any symplectic manifold, the family of symplectic connections is an affine space modelled on the space of symmetric 3-tensors on the manifold. This infinite dimensional space is itself endowed with a symplectic structure and the action of the Lie algebra of Hamiltonian vector fields has a moment map. I shall recall links between the value of the moment map and properties of the associated star product. Selection of symplectic connections via properties of their curvature gives strong rigidity results; manifolds carrying Ricci-type connections can be obtained by reduction of a flat connection.

- Manuel de León (Madrid, Spain)

- Ruben Louis (Urbana-Champaign, United States)

*Lie-Rinehart algebras and Poisson algebras over  $C^\infty$ -rings.*

This is joint work with E. Lerman.

We study Lie-Rinehart algebras in the context of differential geometry. A fundamental question arises when one attempts to relate vector bundles and modules. By the Serre-Swan theorem, the module of sections of a vector bundle  $E \rightarrow M$  is a finitely generated projective module over  $C^\infty(M)$ . Conversely,

every finitely generated projective module over  $C^\infty(M)$  is the module of global sections of some vector bundle  $E \rightarrow M$ . However, it is not clear how to interpret geometrically those modules that are not projective or not finitely generated. Such modules appear, for instance, in the study of singular foliations.

Closely related to this issue is the correspondence between Lie algebroids and Lie-Rinehart algebras. Recall that any Lie algebroid  $E \rightarrow M$  over a smooth manifold  $M$  induces, at the level of sections, a Lie-Rinehart algebra

$$p : \Gamma(E) \longrightarrow \text{Der}(C^\infty(M)).$$

On the other hand, there exist Lie-Rinehart algebras that arise naturally (for example in Poisson geometry), particularly in situations involving singularities, whose underlying modules are not modules of sections of vector bundles (that is, they are not finitely generated projective modules).

In this talk, we address these questions using the framework of  $C^\infty$ -rings, in which the category of smooth manifolds (and more generally that of Poisson manifolds) embeds fully faithfully. This framework provides a natural geometric interpretation of arbitrary modules over  $C^\infty$ -rings. Moreover, it allows to establish results that cannot be obtained using commutative algebra alone.

We introduce Lie-Rinehart algebras over  $C^\infty$ -rings and show that, for any Poisson  $C^\infty$ -ring  $\mathcal{A}$ , the module  $\Omega_{\mathcal{A}}^1$  of  $C^\infty$ -Kähler differentials naturally carries such a structure. Conversely, given a Lie-Rinehart algebra over  $\mathcal{A}$ , we construct a natural Poisson bracket on the  $C^\infty$ -ring  $\mathcal{F}(\mathcal{M})$  associated with the underlying module  $\mathcal{M}$ , and this bracket in turn determines the Lie-Rinehart algebra structure on  $\mathcal{M}$ .

In the case where  $\mathcal{A}$  is the  $C^\infty$ -ring of smooth functions on a smooth manifold  $M$  and  $\mathcal{M} = \Gamma(E)$  is the module of sections of a Lie algebroid  $E \rightarrow M$ , the  $C^\infty$ -ring  $\mathcal{F}(\Gamma(E))$  is the ring of smooth functions  $C^\infty(E^*)$  on the total space of the dual vector bundle  $E^* \rightarrow M$ .

Several examples will illustrate these constructions.

- Mohsen Masmoudi (Nancy, France)

*Leibniz infinity morphisms for derived brackets*

This is a joint work with Camille Laurent-Gengoux. Let  $\mathcal{F}$  be a Lie infinity morphism between two differential graded Lie algebras  $\mathfrak{g}$  and  $\mathfrak{g}'$ . Let  $\alpha$  be a Maurer-Cartan element of  $\mathfrak{g}$  and let  $\beta$  denote its image by  $\mathcal{F}$ . It is known that

the derived bracket of  $\alpha$  (resp.  $\beta$ ) defines on  $\mathfrak{g}[1]$  (resp.  $\mathfrak{g}'[1]$ ) a differential graded Leibniz algebra structure. We construct explicitly a Leibniz infinity morphism between  $\mathfrak{g}[1]$  and  $\mathfrak{g}'[1]$ . As an application of this construction, we recover a formula of Dominique Manchon about the commutator of the star-product.

- Leonid Ryvkin (Lyon, France)

*Darboux theorems in multisymplectic geometry*

A very useful property of symplectic manifolds is that they admit a Darboux theorem: locally any symplectic form looks like the constant coefficient standard model  $\omega = \sum dx_i \wedge dy_i$ . In multisymplectic geometry, i.e. for closed, non-degenerate differential forms of higher degree, both aspects of the Darboux theorem become more subtle:

- (i) There is no unique standard linear model for them.
- (ii) Even fixing the linear model does not assure a constant coefficient representation in general.

In this talk, we are going to give an overview of Darboux theorems in the multisymplectic context and an introduction to the tools used to understand and prove them.

- Friedrich Wagemann (Nantes, France)

*Enhanced Leibniz algebras*

This is joint work with Thomas Strobl (Lyon). Enhanced Leibniz Algebras (ELA) are particular algebraic structure which turned up in non-abelian gauge theory. Roughly speaking, an ELA is a linear map  $t : W \rightarrow V$ , where  $W$  is a linear space and  $V$  a Leibniz algebra together with a product:  $V \times V \rightarrow W$  with some axioms. Our main two results in this work are a structure theorem for ELAs and the existence of a  $\mathcal{L}_\infty$  algebra associated to each ELA.

- Marco Zambon (Leuven, Belgique)

*Reduction of Courant algebroids via graded manifolds*

Courant algebroids are certain objects in Lie theory that are used to define, for instance, Dirac structures and generalized complex structures. We will use the correspondence between degree 2 symplectic manifolds and Courant algebroids to approach the reduction of Courant algebroids using graded geometry. For this purpose we will consider both graded coisotropic submanifolds and a

graded version of moment maps. The resulting reduction procedure, in a particular case, recovers the work of Bursztyn-Cavalcanti-Gualtieri around 2007. This talk is based on joint work with Bursztyn, Cattaneo and Mehta.

## 4 Abstract of the colloquium

Mauro Spera (Brescia, Italy)

*Symplectic and topological aspects of classical and quantum hydrodynamics*

René Descartes, in its (unfinished) “Règles pour la direction de l’esprit” (règle quatrième), 1628, possibly gave one of the clearest explanations of the “essence” of mathematics:

“... j’ai découvert que toutes les sciences qui ont pour but la recherche de l’ordre et de la mesure, se rapportent aux mathématiques, qu’il importe peu que ce soit dans les nombres, les figures, les astres, les sons ou tout autre objet qu’on cherche cette mesure, qu’ainsi il doit y avoir une science générale qui explique tout ce qu’on peut trouver sur l’ordre et la mesure, prises indépendamment de toute application à une matière spéciale, et qu’enfin cette science est appelée d’un nom propre, et depuis longtemps consacré par l’usage, savoir les mathématiques ...”

And within Mathematics, Geometry, ramifying in turn into several different domains, can be viewed as a particular sort of mathematical “esprit” taking inspiration from reality and coming back to it after building up “local” models thereof through investigation of highly abstract “spaces”, far away from direct intuition, but giving rise to a virtuous circle.

This is, in particular, the case of Symplectic Geometry: stemming from the common origin of geometrical optics and analytical mechanics discovered by William Rowan Hamilton, it turned out to provide a unified basis of many theories, also underlying innumerable applications. In the sequel, we shall discuss the specific examples provided by classical hydrodynamics and quantum mechanics in the Madelung formulation, pointing out its possible relevance in knot theoretical issues.

Specifically, the present talk surveys past and recent work on the intermingling of knot theoretic and (multi)symplectic aspects in classical fluid mechanics and in the Madelung-Bohm approach to quantum mechanics. A Clebsch portrait for the

Schrödinger equation will be outlined [S. LNCS (2023), Barbieri & S. EPJP (2024)] building on the regular character of the probability current (turning out to be a momentum map) in contrast to that of the Madelung velocity.

A semiclassical interpretation of the HOMFLYPT polynomial will be presented [Miti & S. BUMI (2021)] building on the Liu-Ricca hydrodynamical approach to the latter and on the symplectic interpretation of framing developed in Besana & S. [JKTR (2006)], based on the geometry of the so-called Brylinski's manifold of (mildly singular) knots (or links) in 3-space.

Time permitting, we shall also review the construction of a homotopy comomentum map à la Ryvkin-Wurzbacher-Zambon (RWZ) [JAMS (2020)], transgressing to the standard hydrodynamical comomentum map of Arnol'd, Marsden and Weinstein and others, duly discussed in the first part of the talk, and we shall finally reinterpret the (Massey) higher order linking numbers in terms of conserved quantities within the RWZ multisymplectic framework and determine knot theoretic analogues of first integrals in involution [Miti & S. JAMS (2022)].

Technicalities will be kept to a minimum for the benefit of a wider audience, with main focus placed on the overall geometrical picture.

## 5 Abstracts of the PhD students' talks

- Sacha Amiel (Lyon, France)

*The symmetry groupoid of a Lagrangian*

In this talk, we first introduce Lie groupoids encoding local symmetries of vector bundles (typically on the fibers) and non-local ones (typically on the base). This allows to recover usual symmetries of Riemannian, conformal, symplectic manifolds and many other ones.

Then, given a Lagrangian, we define its (maximal) symmetry groupoid and show it is a Lie groupoid for polynomial Lagrangians. For scalar field theory, we give a complete classification depending on the polynomial interaction potential.

- Ruben Izquierdo-López (Madrid, Spain)

*On the theory of  $G$ -structures applied to multisymplectic manifolds*

Multisymplectic geometry provides a geometric framework to express the equations of motion of classical field theories, in analogy with how symplectic geometry is used in classical mechanics. A multisymplectic form encodes key

structures of a given theory, including symmetries, conserved quantities, and the Poisson bracket.

In the study of these structures, the use of adapted (or Darboux) coordinates is of central importance. However, unlike in symplectic geometry, the closedness of a multisymplectic form is not sufficient to guarantee the existence of flat coordinates. Indeed, there are examples of closed forms of constant linear type that are not flat, such as those arising in  $G_2$ -structures.

These issues can be understood in a unified way through the theory of  $G$ -structures. In this talk, I will introduce the basic notions of  $G$ -structures and structure tensors. I will then explain how this framework can be used to construct forms whose integrability depends strictly on conditions of order  $k$ , for arbitrary  $k$ . Finally, if time permits, I will discuss some possible directions and ideas related to integrability.

- Deniz Yeral (Dijon, France)

*Open modular functors from non-finite categories*

An open modular functor is a system of representations of mapping class groups of surfaces that are compatible with the gluing along marked intervals. The motivation for this notion comes from two-dimensional conformal field theory. In this talk, I will give the construction of an open modular functor that is given in terms of factorization homology of surfaces. This construction applies to some categories of modules of vertex operator algebras that are non-finite, but rigid.